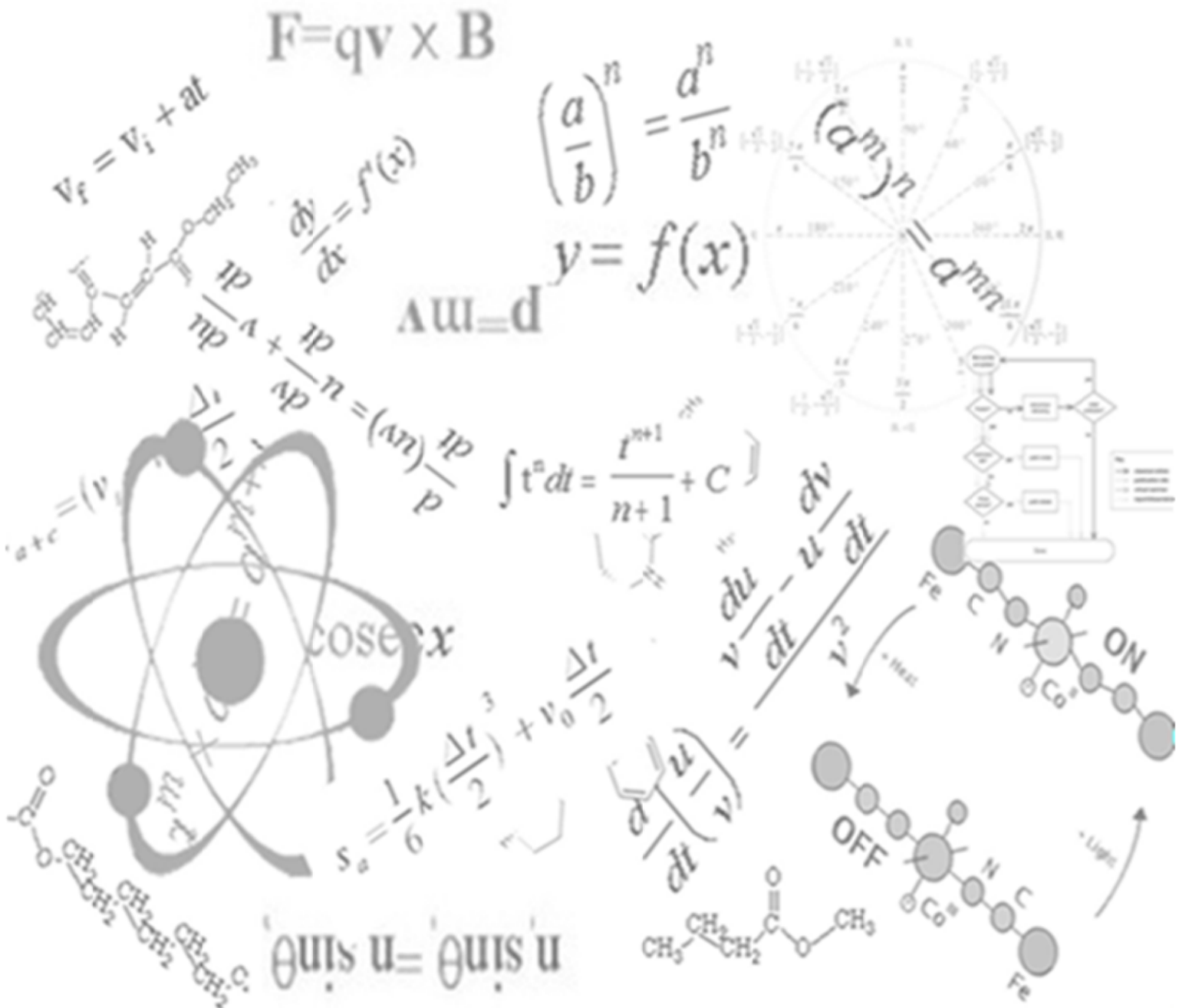


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## Year 12- Mathematics Advanced Applications of Calculus to the Physical World



# Applications of Calculus to the Physical World

## Exam /38

**1. (4 marks)**

The number of bacteria in a colony is growing at a rate proportional to the current number. Find  $k$ , if the size of the colony doubles every half hour. If the colony now has 600 million bacteria, how long ago did the colony contain 3 million bacteria?

**2. (4 marks)**

Water is pumped out of a 25000 litre storage container such that  $\frac{DV}{dt} = -1.92t$ . Find an expression for  $V$  if the tank was initially full. How long before the tank is only 40% full?

**3. (5 marks)**

A particle is moving along a plane and  $x=70t+100e^{-\frac{t}{10}}$ . Find the initial position, show that it is always moving towards the right and find out what happens to the acceleration eventually.

**4. (3 marks)**

Volume of water in a tank is  $V=400-4t+\frac{t^2}{400}$  litres,  $t$  in minutes.

i) how much water is in tank after 15 mins?

ii) what is the rate of water flowing out of the tank at 25 mins?

**5. (5 marks)**

Ice is melting at a rate proportional to its mass,  $\frac{DM}{dt} = -kM$ , after 25 minutes, 15% has melted.

i) show that  $M=M_0e^{-kt}$  is a solution to  $\frac{DM}{dt} = -kM$

ii) find  $k$

iii) how long will it take for 70% to melt?

**6. (2 marks)**

The population of a species of insects is decreasing at an increasing rate. What does this statement imply about  $\frac{DN}{dt}$  and  $\frac{D^2N}{dt^2}$

**7. (3 marks)**

The expected amount of physical activity per day for a student of  $A$  minutes is given by  $A=20e^{kt}$ , where  $t$  is the year the student is in. in year 7, students are expected to spend 120 mins doing activity per day.

i) Find  $k$  (3 dec. plc)

ii) find the rate of the expected amount of time on physical activity is spent on by a student in year 10

**8. (4 marks)**

A particle B is moving along a plane, its position is given by  $x = e^{\frac{t}{4}} - t$

- i) find an expression for  $V$
- ii) find the direction the particle is moving in initially
- iii) find when the particle comes to rest
- iv) find an expression for  $a$ .

**9. (3 marks)**

The height of a specific tide can be expressed as:

$$H = 4 + 2\cos\left(\frac{\pi t}{6}\right)$$

- i) find the rate of change of the height after 10 hours
- ii) is the tide ingoing or outgoing after 10 hours? Give reasons.

**10. (5 marks)**

Given  $v = e^{3t} - 4e^t$  m/s, find:

- i) initial velocity and acceleration
- ii) Find an expression for  $x$  in terms of  $t$  if initially the particle is at  $x=0$
- iii) find the position of the particle at  $t=\ln 4$