

where students come first!



Year 12- Mathematics Advanced
Geometrical applications of
Differentiation

A collage of various mathematical and scientific diagrams and formulas, including:

- $F = qv \times B$
- $V_f = V_i + at$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
- $\frac{dy}{dx} = f'(x)$
- $y = f(x)$
- $\Delta u = d$
- $\int t^n dt = \frac{t^{n+1}}{n+1} + C$
- $\frac{d}{dt} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dt} - u \frac{dv}{dt}}{v^2}$
- $\frac{d}{dt} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dt} - u \frac{dv}{dt}}{v^2}$
- $s_a = \frac{1}{6} k \left(\frac{\Delta t}{2} \right)^3 + v_0 \frac{\Delta t}{2}$
- $\frac{d}{dt} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dt} - u \frac{dv}{dt}}{v^2}$
- $\theta \sin u = \theta \sin' u$
- Diagram of a Bohr-style atom with a central nucleus and three elliptical orbits containing electrons.
- Diagram of a circular sector with radius r and angle θ , showing the arc length $s = r\theta$ and area $A = \frac{1}{2} r^2 \theta$.
- Flowchart with decision diamonds and rectangular process boxes.
- Chemical structures including a carboxylic acid chain, a complex organic molecule with Fe, C, N, O, and ON groups, and a methyl ester CH3-CH2-C(=O)-O-CH3.

Geometrical Applications of Differentiation Exam

/37

1. (2 marks)

Find the primitive function of $y' = 2 + \sqrt{x}$

2. (3 marks)

The second derivative of a function is $y'' = 3x^2 - 2x + 1$, at $x=1$, the corresponding y -value is 0 and the first derivative is equal to 1. Find the equation of the function.

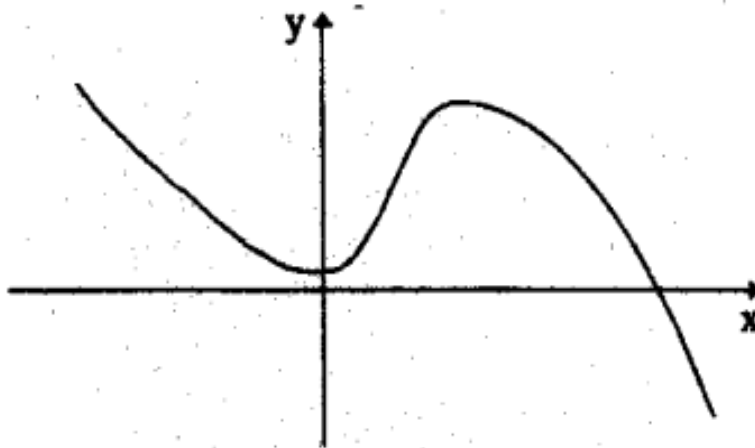
3. (8 marks)

Consider the curve $y = x^3 - 6x^2 + 9x + 1$.

i) find the coordinates of any stationary points and determine their nature

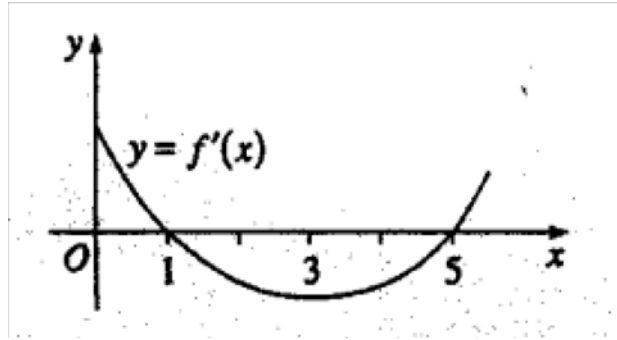
ii) sketch the curve.

4. (2 marks)



The graph shows a sketch of $f(x)$. On the same set of axis draw a sketch of $f'(x)$,

5. (3 marks)



For what value of x does $f(x)$ have a minimum turning point and Find where $f(x)$ has a point of inflexion and why.

6. (9 marks)

For the curve $y=x(x-4)^2$

- i) find any stationary points and determine their nature
- ii) find the point of inflexion
- iii) Draw the graph showing all important features.
- iv) for what values of x is y concave down for $-1 \leq x \leq 5$?

7. (2 marks)

Find an equation for the tangent to the curve $y=\frac{\sqrt{x}}{2}$ at $x=1$.

8. (3 marks)

Find an equation for the tangent to the curve $y=\sqrt{(6-2x)^3}$ at $x=2$.

9. (2 marks)

Given the data:

when $x < 2$, $f'(x) < 0$ and $f''(x) > 0$

when $x = 2$, $f'(x) = 0$ and $f''(x) = 0$

when $x > 2$, $f'(x) < 0$ and $f''(x) < 0$

draw a sketch of $y=f(x)$ given that $f(2)=0$

10. (3 marks)

Given that the area of a rectangle is $A=12x-\frac{x^2}{2}$, find the dimension of this rectangle so that its area is a maximum, where its length is twice its width.